International Journal of Scientific & Technical Development
(A Peer-Reviewed Bi-Annual Journal) ISSN: 2348-4047

CROSSOVER AND MUTATION OPERATORS FOR REAL CODED GENETIC ALGORITHMS

***Shivani Sanan**
Research Scholar, Department of Mathematics, Desh Bhagat University (Punjab) India 147301
Corresponding Author Email: shivanisanan@gmail.com

Abstract
Nature-inspired optimization algorithms have received more and more attention from researchers due to their several advantages. Genetic algorithm (GA) is one of such bio-inspired optimization techniques, which has mainly three operators, namely selection, crossover, and mutation. Several attempts had been made to make these operators of a GA more efficient in terms of performance and convergence rates. One of the critical stages in the genetic algorithm is the crossover process. The crossover operator is believed to be the main search operator in the working of a genetic algorithm (GA) as an optimization tool.A number of crossover operators exist in the GA literature; however, the search power to achieve both of the above aspects differs from one crossover to another. In genetic algorithm (GA), mutation is one of the most important operators responsible for maintaining diversity in the population. The objective of the present study is to introduce a newly designed crossover operator called Logistic Crossover which uses Logistic Distribution. In a genetic algorithm (GA), the mutation is one of the most important operators responsible for maintaining diversity in the population. For a real-coded genetic algorithm (RCGA), this mutation operator is applied variable-wise. In this study, a new positional exponential mutation operator has been proposed for an RCGA. The locations of mutated solutions are made biased to the said information of the problem with the higher probability value. Positional exponential mutation operator, a novel, simple, and efficient real-coded genetic algorithm (RCGA) is proposed and then employed to solve complex function optimization problems. For a real-coded genetic algorithm (RCGA), this mutation operator is applied variable-wise.

Keywords: Evolutionary Algorithms, Genetic Algorithm, Crossover Operator,Mutation Operator, Real Coded

INTRODUCTION

Optimization signifies to procuring the leading solution in a solution space with reference to some pre-established criteria in a decision-making problem. An optimization problem can be defined which optimizes a given function, the norm which is maximized or minimized refers to objection function, the parameters optimizing a given function are called decision variables and the restrictive condition to which parameters are subjected are called constraints.

The basic optimization problem is mathematically formulated as

Minimize/Maximize (Objective function)

$$Z = F(x)$$

subject to constraints

$$g_i(x) \geq or \leq b_i \; ; \; x \geq 0$$

The optimization problems can be categorized in two different categories as mentioned below:

(a) Linear Optimization Techniques:

The term linear programming indicates that the objective function is linear and all the constraints are linear inequalities or equalities. A linear programming problem for optimization techniques can be defined as

Max or Min $Z = CX$

s. t. constraints $AX = B \; ; \; X \geq 0$

(b) Non- Linear Optimization Techniques:

Non-linear optimization generally refers to that problem where the objection function inclines to become non- linear or one or more constraint inequalities possess non-linear relationship or there may be the case where both the conditions apply. In matrix notation, the basic non-linear programming problem can be formulated as follows:-

Max. (or Min.) $Z = F(x)$

subject to the constraints $g_i(x) \geq or \leq b_i; \; x \geq 0$

$$i = 1,2,\ldots\ldots,m; \; x \geq 0$$

where either the objective function $F(x)$ or some $g_i(x)$ or both are non-linear real valued functions of in Genetic algorithms are computerized optimization algorithms that rely on the mechanism of natural genetics to solve both unconstrained and constrained non-linear optimization problems Genetic algorithms attain more significance because of its adaptative behaviour that is it can adjust itself under stochastic conditions and are found to be potential optimization algorithms for complex engineering optimization problems. The need is to obtain a solution in a search space, $S \subset X$ satisfying all the constraints such that $f(x)$ is global minimum or maximum in S .

Genetic algorithms are non-traditional algorithms with few primary differences with almost all conventional optimization methods that these algorithms use coding of variables instead of using variables directly, a population of points in place of a single point and probabilistic operators instead of predefined and determined operators. They are applicable in solving most of the problems that cannot be solved easily by conventional optimization techniques. The global optimum of a multi - modal function cannot be anticipated by classical optimization methods at every simulation. Global optimization plays a vital role if the function space has a number of optimum values of which few values are local and few are global. Traditional optimization methods alone may not be sufficient to solve such problems which involve multiple optima thus paving a way for Genetic Algorithms. Genetic algorithms are based on the biological theory of evolution devised by Charles Darwin in the mid-19th century and provides an extended description to the theory of survival of the fittest. For a specific optimization problem, its feasible solution corelates with the sections of a peculiar class where the fitness of each member is evaluated by the value of the objective function. Instead of working on a single trial solution at a time, GA's process an entire population of trial solutions. As a global search technique, Genetic algorithms work effectively, however they could usually take a somewhat extensive duration to arrive at a global optimal value. Local solution search methods can be integrated with Genetic Algorithms for their smooth conduct and functioning. Genetic Algorithms can be smoothly hybridized with other optimization methods to upgrade its execution.

To make Genetic Algorithms more efficient, several crossover schemes had been founded by various researchers. From the literature survey, it has been observed that there exist numerous crossover techniques with their intrinsicmerits and demerits. The novel concept of crossover is presented in a more effective way in this study. For solving Real Coded Genetic Algorithms, several researchers had focused on evolving diverse mutation operators. It can be said that aexclusive performance of an algorithm on a particular set of problems does not assure an equal execution in solving all other problems. This fact stimulates the researchers to contribute by either refining the existing algorithms or recommending new kind of algorithms. Moreover, this statistic is also the foundation of inspiration for the present study, where a new logistic crossover and positional exponential mutation has been proposed and combined for a real-coded genetic algorithm. It is to be noted that the present work is quite different from the previous work of the authors related to crossover and mutation operation.

In our work, we apply Current optimum opposition. In this opposition scheme, the opposite point is in the vicinity of the global optimal solution all through the course of progression, specifically in advanced phases. Let $x(x_1, x_2, ...x_d)$ be a point in d-dimensional search space, $x_{best}(x_1^{best}, x_2^{best}, ..., x_d^{best})$ be the best solution in the current population and $x_i, x_i^{best} \in [a_i, b_i]$; i=1,2,….,d. The current optimum opposite point of X is denoted by $x^{coo}(x_1^{coo}, x_2^{coo}, ..., x_d^{coo})$ where $x_i^{coo} = 2x_i^{best} - x_i$; i= 1,2, …, d. Here is the center of opposition. There is a probability of the opposition candidate taking a leap and jumping out of the box constraint . In such cases, the opposition candidate is assigned a value as follows: $x_i^{coo} = \begin{Bmatrix} a_i ; x_i^{coo} < a_i \\ b_i ; x_i^{coo} > b_i \end{Bmatrix}$

After applying the Current opposition mechanism on an initial population, the union of both the data set values is taken and the optimal individuals with the fittest solution vector are moved further for processing and operations.

2. Logistic Crossover

The crossover operator applied here is Logistic Crossover. Logistic distribution is a continuous distribution function where its PDFs and CDFs are usually employed in various diverse fields such as logistic regression, logit models, neural networks. It is extensively used in physical sciences, sports modeling, finance, agriculture, etc. The logistic distribution has extensive tails than a normal distribution consequently it is more reliable with the fundamental statistics and delivers an improved perception into the probability of happening of extreme events. The probability density function for the distribution is

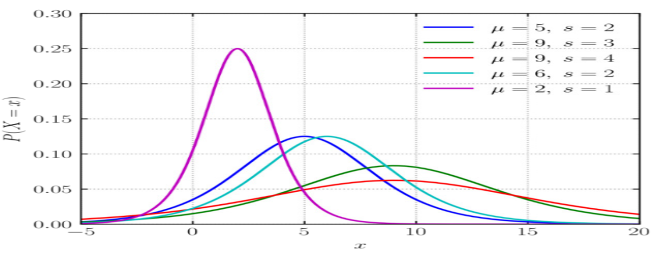

Fig. 1 Probability density function of Logistic distribution

$$f(x;\mu,\sigma) = \frac{e^{-\frac{(x-\mu)}{\sigma}}}{\sigma\left(1+e^{-\frac{(x-\mu)}{\sigma}}\right)^2} \qquad -\infty < x < \infty.$$

The cumulative density function of the distribution is

$$F(x;\mu,s) = \frac{1}{1+e^{-\frac{x-\mu}{s}}}$$

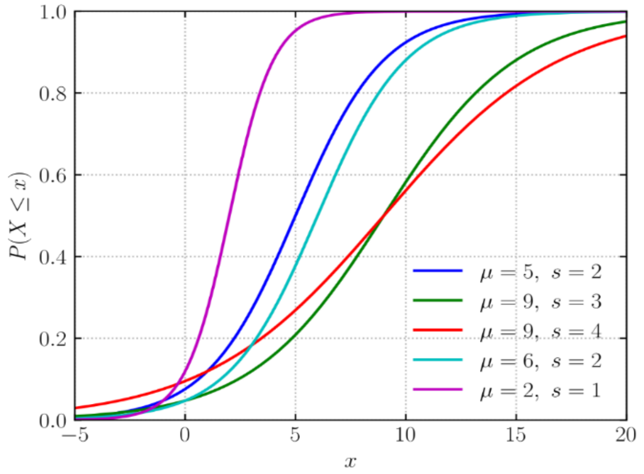

Fig. 2 Cumulative density function of Logistic Distribution

$$\frac{1}{U} = 1 + e^{\frac{-(x-\mu)}{s}}$$, so that

$$x = \mu + s\log\left(\frac{U}{1-U}\right)$$, where U is the uniformly distributed random number between 0 and 1.

To use Logistic Distribution, two parents p1 and p2 are taken to produce two off springs y1 and y2 in the following equations:

y1 = p1* log(x) + p2 * (1-log(x))

y2 = p2 * log(x) + p1 * (1-log(x))

3. Positional Exponential Mutation Operator

For solving Real Coded Genetic Algorithms, several researchers had concentrated on evolving diverse mutation operators. In this paper, a new mutation operator, namely **Position-based Exponential Mutation (PEM)** has been proposed. The name of the proposed mutation operator recommends that it is directed by the positional information of the variables. The positional statistics of a problem supports the algorithm to arrive at global optimum solutions effortlessly. The degree of accuracy of this data serves in obtaining the globally optimum results. As the iteration count proceeds, then the probability of acquiring the expected global optimum solution is more if the positional information of the variables is gathered accurately. In this paper, we have proposed an approximation technique to collect this information during the progress of the solutions. By employing the recommended process, the following steps are depicted for locating the desired information for the variables:

Step 1: At the beginning of each iteration, the average value of all the variables existing in the population is evaluated.

Step 2: At this point, the evaluated average value of the variable is coordinated with its current value already existing in the population solution. At this instant, if the average value of the variable is found to be greater than the value of the parent solution, then the mutated solution proceeds positively in the direction of the parent solution. Else, this information leads to its negative counterpart.

The **Positional Exponential Mutation** operator is operated variable-wise and self-reliantly with a well-defined mutation probability pm.

We compute two different mutation perturbation values ($\beta_1 and \ \beta_2$) using the following equations:

$$\beta_1 = e^{r^2} \times e^{\left(r-2/r\right)} \ldots\ldots(1)$$

$$\beta_2 = e^{\left(r-r^2\right)} \times e^{\left(r-2/r\right)} \ldots.(2)$$

where r is a non-zero uniformly distributed random number in the interval (0,1).

A mutated solution y_m is generated from a parent solution y_p depending upon the nature of positional information as follows:

$$y_m = y_p \pm \left(\beta_1 \vee \beta_2\right)\left|y_{pmean} - y_p\right|$$

4. Algorithm:

1. Generate the initial set of uniformly distributed random solutions inside the search arena
2. Compute the objective function value of each solution vector
3. Apply the Current Optimum Opposition-based Learning Mechanism on the initial population within the search space.
4. Select the fittest solution points by taking the union of all the population points.

5. Initialize the iteration count = 0
6. while<
7. for each individual solution
8. Employ the crossover operator between the best individuals and update the solution as defined in the crossover process
9. Compute the objective function value of the updated solution vector
10. Apply mutation operator as described in mutation equationend of for 12. = + 1 end of while

4.1 Results and Discussions

We execute the above introduced crossover and mutation operators on Himmelblau Function
Minimize
Step 1: We generate the initial population randomly as below:

Table 1Initial Random Population

x1	x2	f(x)
4.6925	5.349	959.68
1.355	0.452	105.520
3.120	3.413	65.026
1.455	1.343	70.868
0.041	2.035	88.273
4.358	3.736	265.556
2.422	0.358	42.598
4.164	2.639	97.699

Step 2: Next, we apply current opposition mechanism on the initial population as

Table 2 PopulationCurrent OppositionMechanism

x1	x2	f(x)
1.5475	0	103.7899
4.885	3.618	392.0799
3.120	0.657	12.2615
4.785	2.727	241.103
6	2.035	740.7585
1.882	0.334	75.8169
3.818	3.712	165.4265
2.076	1.431	35.9322

Step 3: Next, we take union of the above generated set of values and the best individuals with the fittest solution are moved further for processing and operations as follows:

Table 3 Union of above generated population points

x1	x2	f(x)
1.5475	0	103.7899
1.355	0.452	105.520
3.120	0.657	12.2615
1.455	1.343	70.868
0.041	2.035	88.273
1.882	0.334	75.8169
2.422	0.358	42.598
2.076	1.431	35.9322

Step 4: Next, we apply crossover operator by taking the parameters 10, s = 2 and uniformly distributed random number U= 2/3 on the above data points and obtain the results as follows

Table 4 Logistic Crossover

x1	x2	f(x)
1.5868	0	101.2481
1.3779	0.4291	104.7802
3.1825	0.5945	12.0765
1.458	1.3402	70.7932
0	2.0857	86.4864
1.9213	0.2947	74.1135
2.4744	0.3056	40.5453
2.0924	1.4146	35.5634

Step 5: Next, we calculate mean of parent population from above obtained crossover points and apply positional mutation scheme by taking r = 0.9 so that we obtain = 0.5993 and = 0.2917 from equations (1) and (2) and obtain the following results

Table 5 Positional Exponential Mutation

x1	x2	f(x)
1.6916	0.2357	90.0465
1.6079	0.5397	88.0333
2.331	0.6568	42.0619
1.64	1.4954	56.2022
1.0558	1.7134	75.8305
2.0180	0.4444	64.9244
2.9015	0.4522	19.6965
2.2906	1.5915	22.0566

5. Conclusions

In this paper, a novel recombination operator, namely

Logistic Crossover and Positional Exponential Mutation has been proposed and executed in a real-coded genetic algorithm (RCGA). The operating principle of the developed crossover operator is governed by the parametric statistics of the optimization problem to be solved. This acquiredfigures help the search procedure to be focused towards the most potential areas of the variable space. In this study, a new mutation operator, namely positional exponential mutation (PEM) has been proposed and implemented with an RCGA. The knowledge of positional information and utilisation of exponential functions in the proposed scheme, facilitate to make the search procedure of an RCGA more effectual. The recommended operators improve the abilities of Genetic Algorithms in searching global optima as well as in achieving convergence by putting together the local directional search scheme and the adaptive random search approaches. Results indicate that the proposed RCGA is fast, accurate, and reliable, and outperforms all the other GAs.

References

[1] Hedar A.R., Allam A.A.,Deabes W.,"Memory based evolutionary algorithms for nonlinear and stochastic programming problems", Mathematics 2019,7,1126, doi:10.3390/ math 7111126.

[2] Bansal J.C., Singh P.K., Pal N.R. "Evolutionary and swarm intelligence algorithms", Springer: Berlin, Germany, 2019.

[3] Emmerich M., Shir O.M., Wang H,"Evolution Strategies. In Handbook of Heuristics",Springer: Cham, Switzerland, 2018; pp. 89-119, doi: 10. 1007/ 978 -3-319- 07124-4-13.

[4] Bogory- Haddad, O.,Solgi M.,Loaiciga H.A.,"Meta-heuristic and evolutionary algorithms for engineering optimization", John Wiley and Sons: Hoboken, NJ, USA, 2017; Volume 294.

[5] Mahmoodabadi M, NematiA,"A novel adaptive genetic algorithm for global optimization of mathematical test functions and real-world problems", Eng. Sci. Technol. Int J. 2016, 19, 2002-2021.

[6] Deep Kusum& Manoj Thakur, "A new crossover operator for real coded genetic algorithms", Applied mathematics and computation 188, no. 1, 2007: 895-911.

[7] Wright AH, "Genetic algorithms for real parameter optimization", In: Rawlins GJE (ed) Foundations of genetic algorithms, vol 1. Elsevier, pp 205–218. https://doi.org/10.1016/B978-0-08-050684- 5.50016-1

[8] Eshelman LJ& Schaffer JD,"Real-coded genetic algorithms and interval-schemata", In: Whitley LD (ed) Foundations of genetic algorithms,1993, vol 2. Elsevier, pp 187–202. https://doi.org/10.1016/ B978-0-08-094832-4.50018-0

[9] Deb K, Agrawal RB, "Simulated binary crossover for continous search space", Complex Syst 9(3):1–15 1994

[10] Ono I, Kita H, Kobayashi S,"A real-coded genetic algorithm using the unimodal Normal distribution crossover", In: Ghosh A, Tsutsui S (eds) Advances in evolutionary computing: theory and applications. Springer Berlin Heidelberg, Berlin,2003, pp 213–237. https://doi.org/10.1007/978-3-642-18965-4_8

[11] Ono I, Kita H, Kobayashi S,"A robust real-coded genetic algorithm using unimodal normal distribution crossover augmented by uniform crossover: effects of self-adaptation of crossover probabilities", In: Proceedings of the 1st annual conference on genetic and evolcomput - volume 1, Orlando, Florida, 1999. Morgan Kaufmann Publishers Inc., San Mateo, CA, pp 496–503

[12] Kita H, Ono I, Kobayashi S,"Multi-parental extension of the unimodal normal distribution crossover for real-coded genetic algorithm", In: Proceedings of the 1999 congress on evolutionary computation, pp 1588–1595. https://doi.org/10.1109/CEC.1999. 782672

[13] Tsutsui S, Yamamura M, Higuchi T,"Multi-parent recombination with simplex crossover in real coded genetic algorithms", In: Proceedings of the 1st annual conference on genetic and evolcomput,1999, volume 1, Orlando, Florida. Morgan Kaufmann Publishers Inc., pp 657–664

[14] Deb K, Anand A, Joshi D,"A computationally efficient evolutionary algorithm for real-parameter optimization",EvolComput2002, 10(4):371–395. https://doi.org/10.1162/106365602760972767

[15] Kuo H-C, Lin C-H,"A directed genetic algorithm for global optimization",Appl Math Comput 2013, 219(14):7348–7364. https://doi. org/10.1016/j.amc.2012.12.046

[16] Lim SM, Sulaiman MN, Sultan ABM, Mustapha N, Tejo BA,"A new real-coded genetic algorithm crossover: Rayleigh crossover", Journal of Theoretical & Applied Information Technology,2014, 62(1):262–268

[17] Das AK, Pratihar DK,"A direction-based exponential crossover operator for real-coded genetic algorithm", Paper presented at the seventh international conference on theoretical, applied, computational and experimental mechanics, 2017, IIT Kharagpur, India

[18] Chuang Y-C, Chen C-T, Hwang C,"A real-coded genetic algorithm with a direction-based crossover operator", Inf Sci, 2015, 305: 320–348. https://doi.org/10.1016/j.ins.2015.01.026